New West Charter High School -- Calculus BC -- Chapter 4 -- 115 points

Be clear and organized in all of your answers. State conclusions if necessary. Show all of your work. Pencils only, please. Simplest answers are best. Pages one thru three today, page four tomorrow.

5 points.

1) Three times a number exceeds four times its cube by a maximum amount. What is that maximum amount?

2) Find the equation of the line that connects the two points (c, f(c)) where the points x = c are within the domain [0, 1] and both satisfy the relationship $f'(c) = \frac{f(b) - f(a)}{b - a}$ for the function $f(x) = -3x^4 + 6x^2 + 4$ in that domain. (15 points)

3) Find the derivative at each critical point and determine the local extreme values for $y = x^{2/3}(x^2 - 4)$. (10 points)

Five points each. Use differentials to estimate the following values. Then show the calculated value to five decimal places.

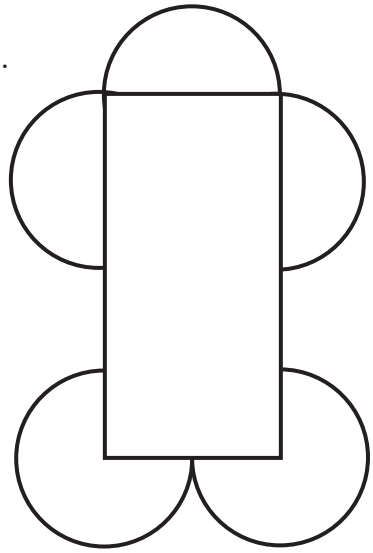
4) cos 58°

5) $(103)^{1.5}$

6) Find the linearization L(x) of f(x) at x = a for $f(x) = x^3 - 2x + 3$ and a = 2. (5 points)

7) You are planning to make an open rectangular box from an 12" by 16" piece of cardboard by cutting congruent squares from the corners and folding up the sides. What are the dimensions of the box of largest volume AND what is its volume? (10 points)

8) A fancy stained glass window in the shape of a rectangle with halves and 3/4 of identical circles on its sides is to be bordered with 120 inches of golden stripping. The bottom corners of the rectangle coincide with the centers of the circles. Find the radius of each circular part if the area is to be a maximum. (10 points)



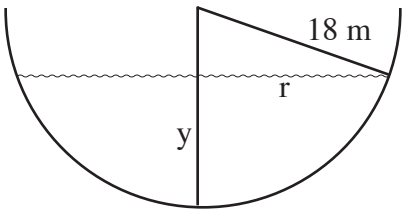
9) The ABCD company can produce up to 60 hippledipples a week. They know that n hipdips (as they are called in the factory) per week can be sold for p dollars each, where $p = 240 - 5n$, and the cost of producing n hipdips is $600 + 18n + n^2$ dollars. How many hippledipples should be produced each week in order to yield the greatest profit? (10 points)

10) A 12-foot ladder is leaning against a wall. The top of the ladder is sliding down the wall at the rate of 1.5 feet per second. (10 points)

a) How fast is the bottom of the ladder moving along the ground at the point in time when the ladder is six feet from the wall?

b) How is the angle that the ladder makes with the ground changing at that same moment?

11) Water is flowing at the rate of 4 m^3/min out of a hemispherical reservoir of radius 18 m. Given that the volume of water in the reservoir is $V = \frac{\pi}{3} y^2 (3r - y)$ when the radius of the water is r meters and it is y meters deep, at what rate is the water level changing when the water is 12 m deep? (10 points)



12) For $f(x) = 1 - x^2 e^{1-x}$, use analytical methods to find: (20 points)

a) f'(x) b) f''(x)

c) the exact intervals where the function is increasing

d) the exact intervals where the function is decreasing

e) the exact intervals where the function is concave up

f) the exact intervals where the function is concave down

g) any local extreme values

h) the x-coordinate of any points of inflection

i) and of course, sketch its graph

Properly label your answers by the letters above if you want credit.