

**New West Charter High School -- Calculus BC Honors -- Exam #2 -- 100 points**

Show all of your work. Partial credit for partial performance. Simplify where practical.  
Seven points each unless specified otherwise.

- 1) Explain, with a proper diagram, how we arrive at the definition for the derivative.  
Include an arbitrary function, secant, tangent, points, labels, and whatever words you need to explain it coherently. (ten pts)

2) For  $f(x) = \frac{x-5}{x+2}$ , find  $f'(0)$

- 3) Determine at which value(s) of  $t$ , the function

$$g(t) = \frac{t}{1+t^2} \text{ has a horizontal tangent.}$$

4) Find  $dy/dx$  for  $f(x) = -3\sin x \cos x$   
when  $x = \pi/6$

- 5) Name the four categories of reasons why a function may not be differentiable everywhere. No essays here.

6) Given the position of a particle, traveling in one dimension, is given by  $x(t) = t^3 - 4t + 3$ , for  $0 \leq t \leq 2$ , find the change in position during that interval, the average speed during that interval, and the velocity of the particle at  $t = 2$  seconds. Ten points.

7) Find the values of  $c$  and  $k$  so that  $f(x)$  is differentiable given

$$f(x) = \begin{cases} 2x^3 - 3cx + k & x < 1 \\ cx^2 + 4k & x \geq 1 \end{cases}$$

8) Find  $dr/d\theta$  for  $r(\theta) = \theta \sin \theta + \theta^{-2} \cos \theta$

9) Find  $\frac{d^{1081}}{dx^{1081}}(\sin x)$

10) The Arrow-Pratt Measure of Relative Risk Aversion ratio in microeconomics is given as  $\frac{-u''(x)}{u'(x)}$ . Given that  $u = \sqrt{x}$ , find that ratio.

11) Use the product rule to show that the derivative of  $\sin^3 x$  is  $3 \sin^2 x \cos x$ .

12) Consider the function  $f(x) = x(x+3)(x-2)$ . The normal line to this function at the point  $x = 1$  intersects the graph of  $f(x)$  at another point in the fourth quadrant. Find the  $x$ -coordinate of that intersection.  
Ten points. Two extra credit points if you can find its  $y$ -coordinate.

13) Last one. At which point(s) of the graph of  $f(x) = 5x^3$  is the tangent line parallel to  $y = 2x - 3$ ?  
At which point(s) is it normal to the line  $y = x + 5$ ?

