New West Charter High School - Honors Calculus BC - Test #1 - 95 points

Show all your work. Partial credit for partial performance. No copying of any kind. Show everything you want considered in a neat and complete manner. If I cannot read it, you don't score.

1) A particle moves in one dimension. The distance in cm it covers in the x direction is governed by the equation (ten points)

$$x(t) = \frac{5}{3}t^3 - 4t^2 - 6t + 3$$

   a) What is the particle's average speed during the first three seconds of its motion?
   b) To the nearest second, when is the particle motionless (aside from $t = 0$)?

Determine the following limits (five points each):

2) $\displaystyle\lim_{t \to 0} \frac{7\sin 8t}{6\sin 5t}$

3) $\displaystyle\lim_{x \to 6} \frac{24 - x}{x^2}$

4) $\displaystyle\lim_{x \to \infty} \frac{6x^2 + 42x - 5}{4 - 18x^2 - 3\sqrt{x^3}}$

5) $\displaystyle\lim_{x \to 0} \frac{-\sin 4x + 3e^{2x}\sin 4x}{4x}$

6) $\displaystyle\lim_{n \to 0} \frac{\frac{1}{2+n} - \frac{1}{2}}{n}$

7) $\displaystyle\lim_{x \to -\pi/6} \left(\frac{\pi}{x}\cos 4x\right)$

7) For ten points, using the limit method we showed in class for finding the slope of a tangent to a curve,
   a) find the slope of the tangent to the graph of the following function, and b) evaluate it at x = 4
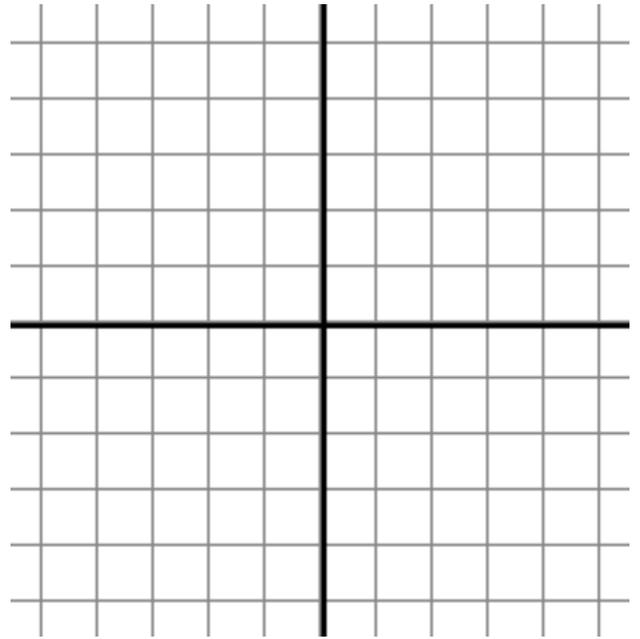   (YOU MUST SHOW WORK!):

$$f(x) = -x^3 + 7x + 2$$

8) Using the fictitious (and silly) function below, discuss the continuity of each interval and discuss the limits
   and values of the function at each integral value of x. Be complete here. Ten points.
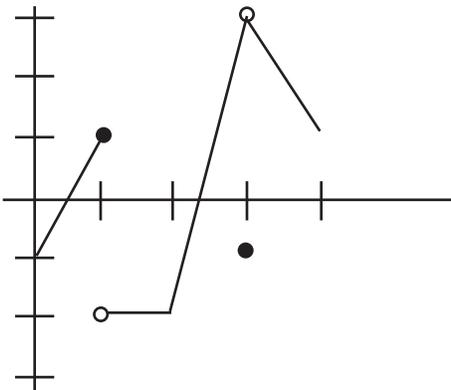
9) For the following function, determine what asymptotes exist analytically (and their location); then graph it (ten pts):

$$f(x) = \frac{x^2 - x + 6}{x + 2}$$

10) For the following function, state appropriate right- AND left-end behavior models. Demonstrate why without graphing. Five points.

$$y = x^2 - \ln(2 + \sin(x^4))$$

For five points each, find and characterize any discontinuities that exist in the following functions:

11) $h(t) = \tan 2t$

12) $\gamma(\kappa) = \dfrac{\kappa^2 - 5\kappa + 4}{\kappa^2 - 1}$

13) For $x \neq 2$, the function g(x) is equal to $\dfrac{x^3 - 4x^2 - 11x + 30}{x^2 - 4}$. What value should be assigned to

g(2) to make g(x) continuous at x = 2? Five points.

14) An object is dropped from the top of a 250-m spaceship on Jupiter. Its height above ground after t seconds is given by $250 - 16t^2$. How fast is it falling 2 seconds after it is dropped? Five points.

15) EXTRA CREDIT: Use either prior knowledge, your analytical skills, or your calculator to evaluate this limit exactly:

$$\lim_{n \to \infty} (1 + \frac{1}{n})^n$$