

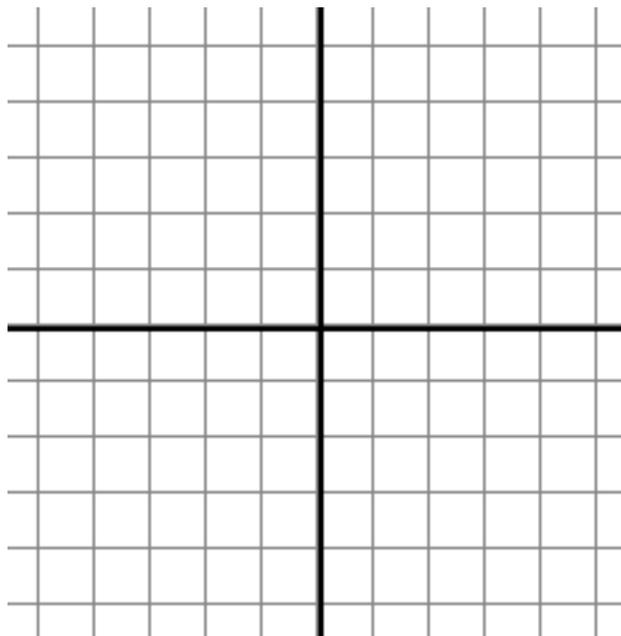
Beverly Hills High School -- AP Calculus BC -- Test #4 -- Chapter 4 -- 100 points

Be sloppy and disorganized. Cheat. Copy. Finagle. Forget to be clear and professional. NOT!!

Partial credit for partial achievement. Decimal answers to 3 places. SHOW ALL YOUR WORK.

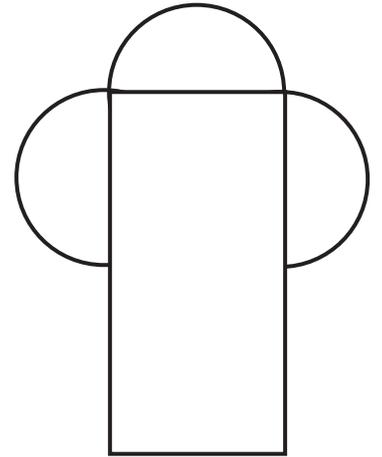
1) For $f(x) = x^3e^{1-x}$, use analytical methods to find: (20 points)

- a) $f'(x)$ b) $f''(x)$
- c) the exact intervals where the function is increasing
- d) the exact intervals where the function is decreasing
- e) the exact intervals where the function is concave up
- f) the exact intervals where the function is concave down
- g) any local extreme values
- h) the x-coordinate of any points of inflection
- i) and of course, sketch its graph



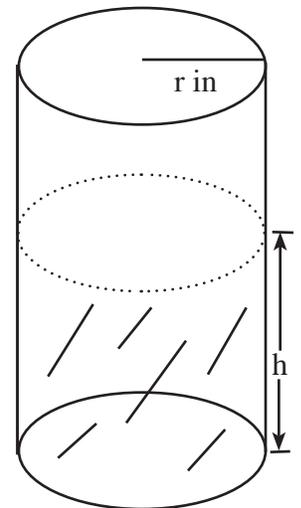
- 2) The XYZ company can produce up to 40 stone gargoyles a week. They know that n gargoyles per week can be sold for p dollars each, where $p = 160 - 3n$, and the cost of producing n gargoyles is $500 + 12n + n^2$ dollars. How many gargoyles should be produced each week in order to yield the greatest profit? (10 points)
- 3) You are planning to make an open rectangular box from an 8" by 12" piece of cardboard by cutting congruent squares from the corners and folding up the sides. What are the dimensions of the box of largest volume AND what is its volume? (10 points)
- 4) Find the equation of the line(s) that run(s) through the point $(c, f(c))$ where the point $x = c$ is within the domain $[0, 1]$ and satisfies the relationship $f'(c) = \frac{f(b) - f(a)}{b - a}$ for the function $f(x) = -2x^3 + 4x^2 + 1$ in that domain. (10 points)

- 5) A fancy stained glass window in the shape of a rectangle with identical semicircles on three sides is to be bordered with 72 inches of golden stripping. Find the radius of each semicircular part if the area is to be a maximum. (10 points)



- 6) A coffeepot has the shape of a right cylinder with radius r inches. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t , measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. The volume of a cylinder is given by $V = \pi r^2 h$. Ten points.

- a) If $r = 5$ inches, find dh/dt .
 b) If instead the radius is unknown but is $1/3$ the height of the coffeepot, find dh/dt when $h = 15$ inches.



- 7) Just set up the equation for finding the roots of the function $f(x) = x^2 + \sqrt{x} - 6$ using Newton's Method, given that a good choice for x_1 is 2 by showing how to get x_2 and x_n . Ten points.

Five points apiece on these.

- 8) Find the linearization $L(x)$ for the function $f(x) = 3 - 2x + x^2$ about the point $x = 1$.

Using differentials, find the following. Then show the calculated value to five decimal places:

- 9) $\sin 28^\circ$

- 10) A trough 20 feet long has a cross section that is an equilateral triangle 12 feet across. If water flows into it at a rate of 8 cubic feet per minute, how fast is the surface rising when the water is 3 meters deep. Ten points. (Hint: $1:2:\sqrt{3}$)